Name: _____

Math 30 Calculus I
Spring 2016

Recall that, $\frac{d}{dx}\ln|x| = \frac{1}{x}$. We will use **logarithmic differentiation** to find the derivative of

$$y = \frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}$$

1. Start by taking the (natural) logarithm of both sides, and simplifying.
   To simplify recall:
   - $\ln\frac{a}{b} = \ln a - \ln b$
   - $\ln ab = \ln a + \ln b$
   - $\ln a^b = b\ln a$.

$$\begin{aligned}
\ln y &= \ln\left(\frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}\right)\\
\ln y &= \ln\left(x^{3/4}\sqrt{x^2+1}\right) - \ln(3x+2)^5\\
\ln y &= \ln x^{3/4} + \ln\left(x^2+1\right)^{1/2} - \ln(3x+2)^5\\
\ln y &= \frac{3}{4}\ln x + \frac{1}{2}\ln\left(x^2+1\right) - 5\ln(3x+2)
\end{aligned}$$

2. Now implicitly differentiate the equation in (1).

$$\left(\frac{1}{y}\right)\frac{dy}{dx} = \left(\frac{3}{4}\right)\frac{1}{x} + \left(\frac{1}{2}\right)\frac{2x}{x^2+1} - 5\frac{3}{3x+2}$$

3. Solve this for $\frac{dy}{dx}$.

$$\frac{dy}{dx} = y\left[\frac{3}{4x} + \frac{x}{x^2+1} - \frac{15}{3x+2}\right]$$

4. Substitute $\frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}$ in for y in the resulting equation.

$$\frac{dy}{dx} = \left(\frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}\right)\left[\frac{3}{4x} + \frac{x}{x^2+1} - \frac{15}{3x+2}\right]$$

Use logarithmic differentiation to take the derivative of the following functions.

1. $y = x^x$

2. $y = \dfrac{x^{1/x}}{x^x}$

3. $y = \dfrac{5x^3 x^{x^2-1}}{4x^3 - 8}$