Name: Answers

Math 35 Linear Algebra
Spring 2016

1. Find the determinant of the following matrix A.

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & -2 \\ 0 & 3 & 1 & -1 & 2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -2 & 3 \\ 1 & 2 & 1 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 & -1 \\ 0 & 3 & 1 & -1 & 2 \\ 0 & 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix}$$

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2. Show that $\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix} = 2v_1 + v_2 - v_3 + 3v_4$

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \\ 4 \\ -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ -1 \end{bmatrix} \right\}$$

v_1 v_2 v_3 v_4

$$\begin{bmatrix} 1 & 2 & 6 & 1 & 1 \\ -1 & 2 & 2 & 1 & 1 \\ 0 & 0 & 4 & 2 & 2 \\ 1 & 0 & -2 & -1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 6 & 1 & 1 \\ 0 & 4 & 8 & 2 & 2 \\ 0 & 0 & 4 & 2 & 2 \\ 0 & -2 & -8 & -2 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 6 & 1 & 1 \\ 0 & 1 & 2 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 4 & 2 & 2 \\ 0 & 0 & -4 & -1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 6 & 1 & 1 \\ 0 & 1 & 2 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 6 & 0 & -2 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 6 & 1 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 6 & 1 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

Refers to A in (1)

3. Determine if the following are true or false and explain how you know

I A is invertible $\det A \neq 0$

T For every vector $\boldsymbol{b}$ in $\mathbb{R}^5$, the augmented matrix $[A\ \boldsymbol{b}]$ has a pivot in every column.

I For every vector $\boldsymbol{b}$ in $\mathbb{R}^5$, the augmented matrix $[A\ \boldsymbol{b}]$ has a pivot in every row.

I $\text{Col}(A) = \mathbb{R}^5$.

4. Determine if the following are subspaces of $\mathbb{R}^2$.

(a) $W = \left\{ \begin{bmatrix} x+3 \\ x-3 \end{bmatrix} : x \in \mathbb{R} \right\}$ No, not closed under scalar mult.

(b) $X = \{\mathbf{0}\}$ Yes $\quad \bar{0} + \bar{0} = \bar{0}$, $c \cdot \bar{0} = \bar{0}$

(c) $Y = \text{span}(t, t^3)$

No, vectors are polynomials

5. Suppose B is an $m \times n$ matrix such that $B\boldsymbol{x} = \mathbf{0}$ has exactly one solution.

(a) How many pivot positions does B have?

n since exactly one solution so a pivot in each column

(b) Is B invertible?

If $m = n$

(c) Can you determine what the **only** solution is?

$\bar{0}$

6. Let $W = \text{span}\left(\begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 6 \\ 2 \\ 4 \\ -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \\ -1 \end{bmatrix}\right)$.

(a) Find a basis for W.

$\begin{bmatrix} 3 & 0 & 6 & 1 \\ 1 & 0 & 2 & 1 \\ 2 & 0 & 4 & 2 \\ -1 & 0 & -2 & -1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 2 & \frac{1}{2} \\ 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 2 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\left\{\begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ -1 \end{bmatrix}\right\}$

(b) What is the dimension of W?

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7. Determine if the following are linearly independent.

$$\left(\begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 6 \\ 2 \\ 4 \\ -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \\ -1 \end{bmatrix}\right)$$

$\left[\begin{array}{cccc|c} 3 & 2 & 6 & 1 & 0 \\ 1 & 2 & 2 & 1 & 0 \\ 2 & 0 & 4 & 2 & 0 \\ -1 & 0 & -2 & -1 & 0 \end{array}\right]$ No, R_2 blanks R_4 so at most 3 pivots

8. Let $\mathcal{A} = \left\{ \boldsymbol{a_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \boldsymbol{a_2} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$ and $\mathcal{B} = \left\{ \boldsymbol{b_1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \boldsymbol{b_2} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$.

(a) Let $\boldsymbol{x} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$. Find $[\boldsymbol{x}]_{\mathcal{A}}$: the $\mathcal{A}$-coordinate vector of $\boldsymbol{x}$.

$$\begin{bmatrix} 4 \\ 0 \end{bmatrix} = a\begin{bmatrix} 1 \\ 1 \end{bmatrix} + b\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -4 \\ 4 \end{bmatrix}$$

(b) Let $\boldsymbol{x} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$. Find $[\boldsymbol{x}]_{\mathcal{B}}$: the $\mathcal{B}$-coordinate vector of $\boldsymbol{x}$.

$$\begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

9. Find a basis for the null space of the matrix A.

Null space is solutions to $A\vec{x} = \vec{0}$, so

$$A = \begin{bmatrix} 1 & 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 2 & 4 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 2 & 4 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccccc|c} 1 & 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccccc|c} 1 & 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

So $x_1 = 0$

$x_2 + 2x_3 = 0$

$x_4 = 0$

$x_5 = 0$

$x_3 = c$ is free variable

$x_1 = 0$

$x_2 = -2c$

$x_4 = 0$

$x_5 = 0$

So basis of nullspace is $\left\{ \begin{bmatrix} 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$